A long thin wire with charge density λ

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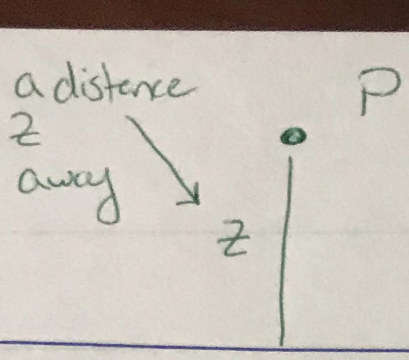
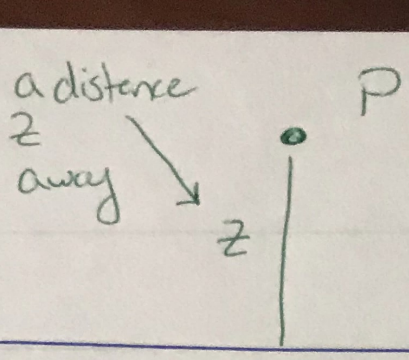
what is $\vec{E}$ at Pt P

A long thin wire with charge density λ



what is $\vec{E}$ at Pt P

A long thin wire with charge density λ



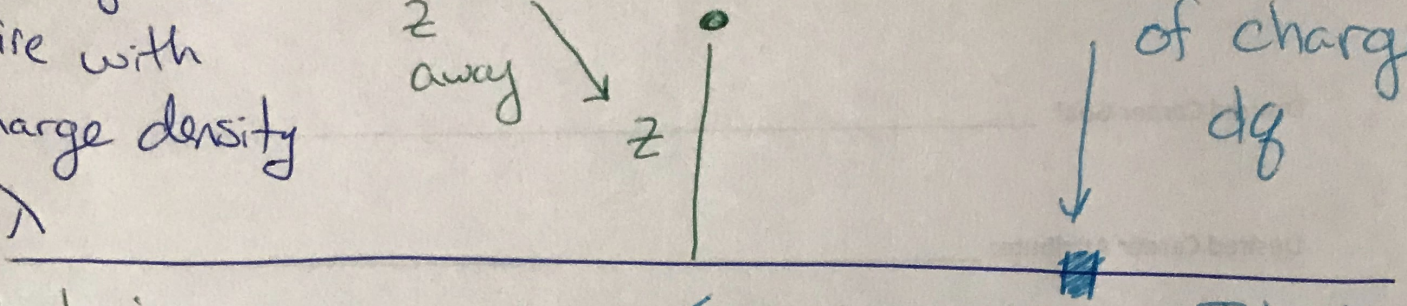
what is $\vec{E}$ at Pt P

A long thin wire with charge density λ

a distance z away

P

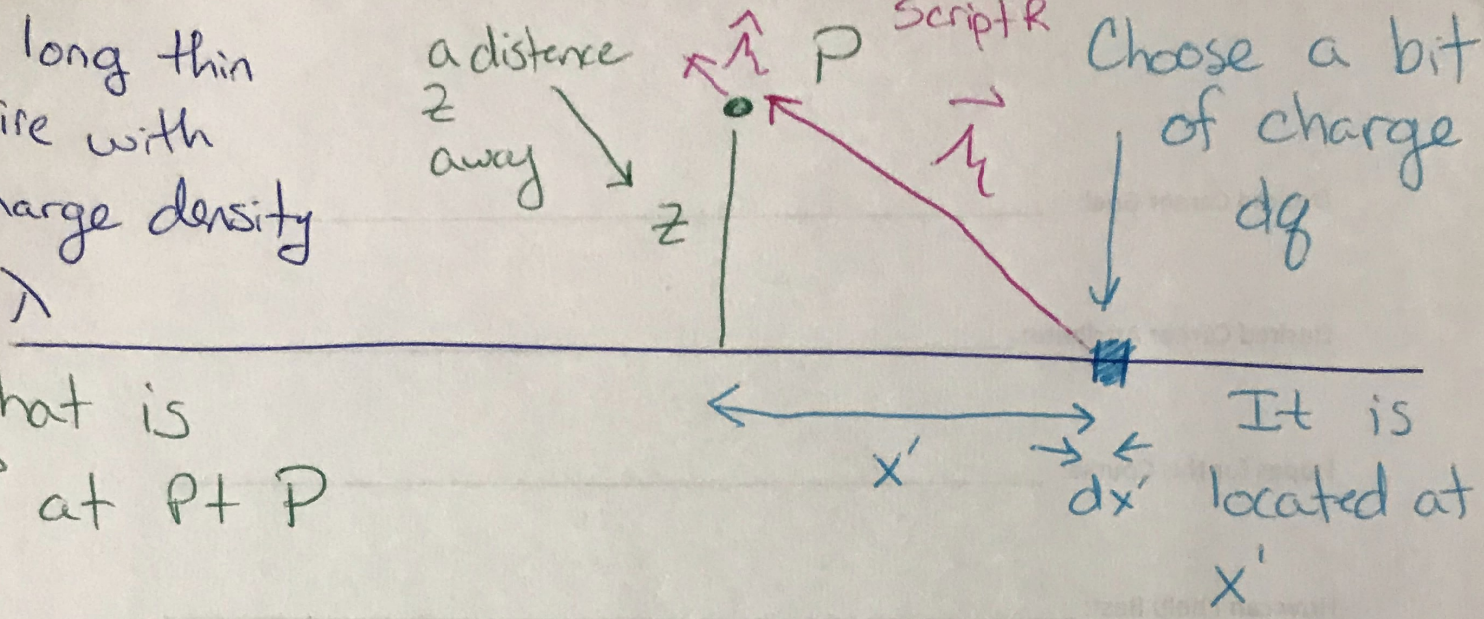
Choose a bit of charge dq



What is $\vec{E}$ at Pt P

It is located at x'

A long thin wire with charge density λ



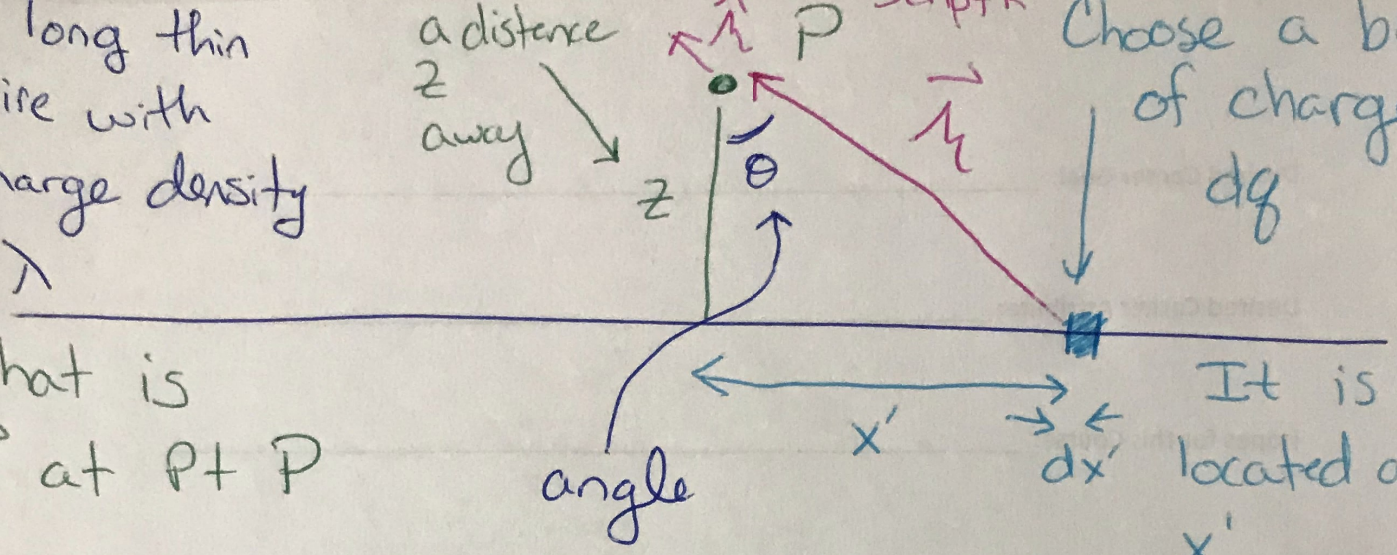
what is $\vec{E}$ at Pt P

A long thin wire with charge density λ

a distance z away

P

Choose a bit of charge dq



what is $\vec{E}$ at Pt P

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A long thin wire with charge density λ

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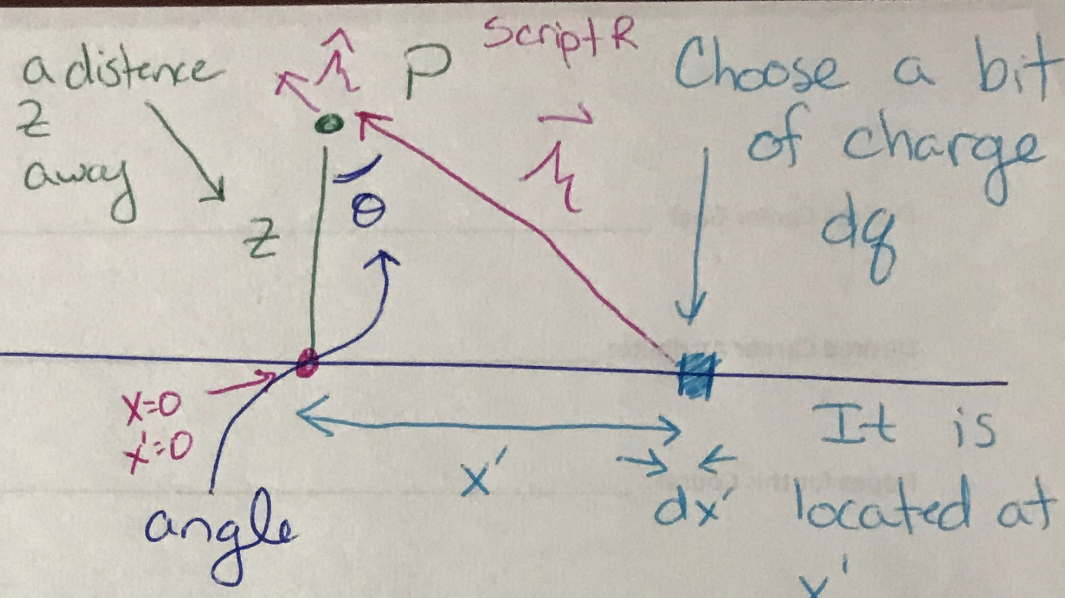


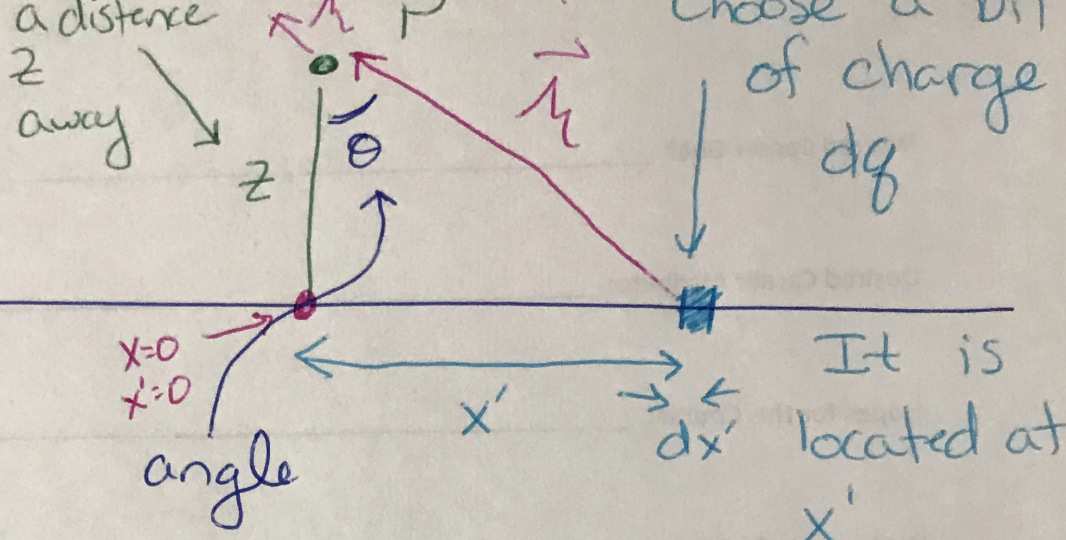
$$|\mathcal{R}| = \sqrt{x'^2 + z^2}$$

$$\vec{\mathcal{R}} = (x-x')\hat{i} + (y-y')\hat{j} + (z-z')\hat{k}$$

$$\vec{\mathcal{R}} = -x'\hat{i} + z\hat{k}$$

A long thin wire with charge density λ

a distance z away



choose a bit of charge dq

It is located at x'

what is $\vec{E}$ at P+ P

$$|\vec{\imath}| = \sqrt{x'^2 + z^2}$$

$$\vec{\imath} = (x - x')\hat{i} + (y - y')\hat{j} + (z - z')\hat{k}$$

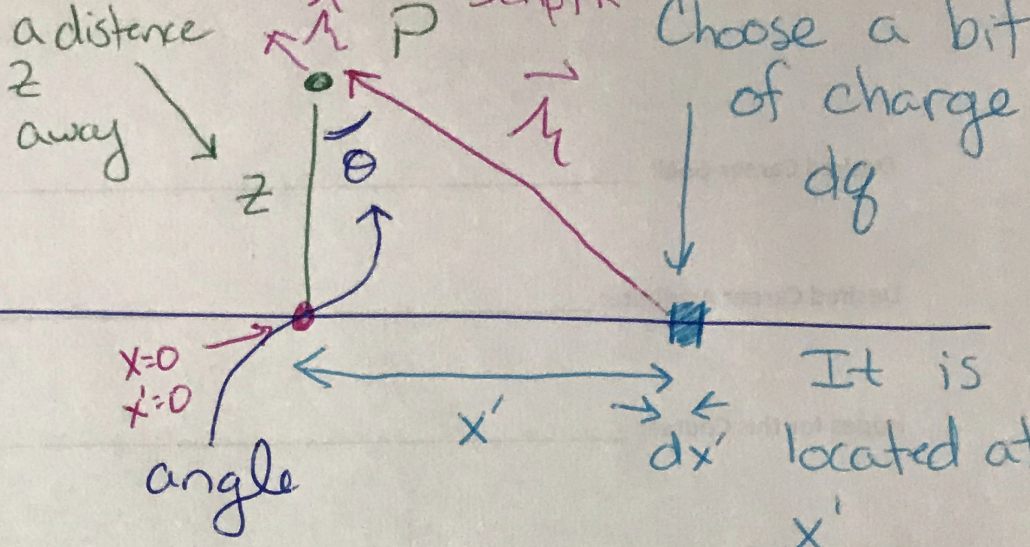
$$\vec{\imath} = -x'\hat{i} + z\hat{k}$$

$$\hat{\imath} = \frac{\vec{\imath}}{\imath} = -\frac{x'\hat{i} + z\hat{k}}{\sqrt{x'^2 + z^2}}$$

$$\hat{\imath} = -\frac{x'}{\sqrt{x'^2 + z^2}}\hat{i} + \frac{z}{\sqrt{x'^2 + z^2}}\hat{k}$$

A long thin wire with charge density λ

a distance z away



what is $\vec{E}$ at P?

$$|\vec{\imath}| = \sqrt{x'^2 + z^2} \qquad \vec{\imath} = (x-x')\hat{\imath} + (y-y')\hat{\jmath} + (z-z')\hat{k}$$

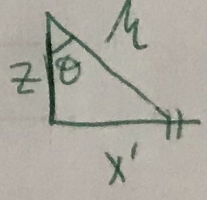
$$\vec{\imath} = -x'\hat{\imath} + z\hat{k}$$

$$\hat{\imath} = \frac{\vec{\imath}}{\imath} = -\frac{x'\hat{\imath} + z\hat{k}}{\sqrt{x'^2 + z^2}}$$

$$\hat{\imath} = -\frac{x'}{\sqrt{x'^2 + z^2}}\hat{\imath} + \frac{z}{\sqrt{x'^2 + z^2}}\hat{k}$$

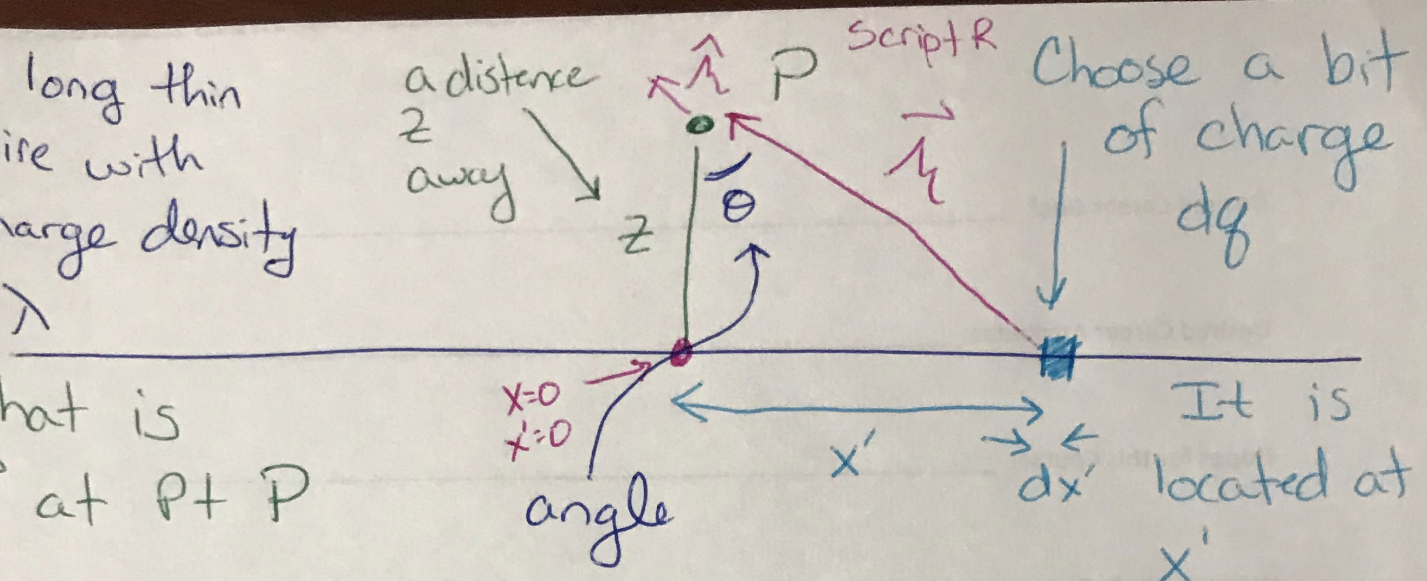
trick

$$\hat{\imath} = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{k}$$



A long thin wire with charge density λ

what is $\vec{E}$ at P+P



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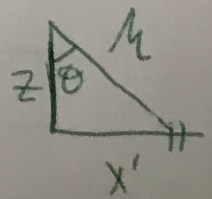

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{\mathcal{R}^2}\hat{\mathcal{R}} = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda\,dx'}{\mathcal{R}^2}\hat{\mathcal{R}}$$

$$\frac{1}{\mathcal{r}^2} \rightarrow \frac{\cos^2\theta}{z^2} \qquad \frac{z}{\mathcal{r}} \rightarrow \cos\theta$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda \cdot \cos^2\theta}{z^2} \cdot \hat{\mathcal{r}}\, dx'$$

$$\frac{1}{r^2} \to \frac{\cos^2\theta}{z^2} \qquad \frac{z}{r} \to \cos\theta$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda \cdot \cos^2\theta}{z^2} \cdot \hat{r}\, dx'$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{\lambda}{z^2} \int \cos^2\theta \left[-\sin\theta\, \hat{i} + \cos\theta\, \hat{k}\right] d$$

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NEED trick

$$\frac{x'}{z} = \tan\theta$$

$$x' = z\tan\theta$$

$$dx' = z \frac{1}{\cos^2\theta}\, d\theta$$

$$\frac{1}{\mathcal{r}^2} \to \frac{\cos^2\theta}{z^2} \qquad \frac{z}{\mathcal{r}} \to \cos\theta$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda \cdot \cos^2\theta}{z^2}\cdot \hat{\mathcal{r}}\, dx'$$

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$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda \cdot \cos^2\theta}{z^2}\cdot \hat{\mathcal{r}}\, dx'$$

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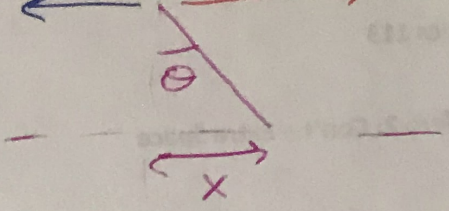


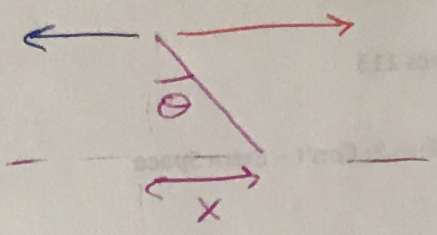
$x \to -\infty$

$\theta \to -\pi/2$

$x \to +\infty$

$\theta \to +\pi/2$

$$\vec{E} = \frac{\lambda}{4\pi\epsilon_0 z}\left\{-\int_{-\pi/2}^{\pi/2} \sin\theta \, d\theta \, \hat{i} + \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta \, \hat{k}\right.$$



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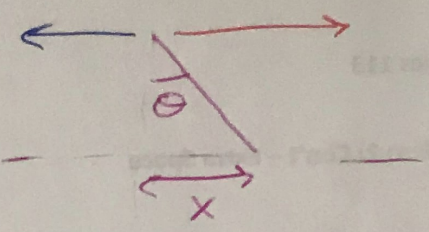
$$\vec{E} = \frac{\lambda}{4\pi\epsilon_0 z}\left\{ 0(-\hat{i}) + 2\hat{k} \right\}$$



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$$\vec{E} = \frac{\lambda \hat{k}}{2\pi\epsilon_0 z}$$