Physics 380-001

Exam #2 PRACTICE EXAM

Your name: ______________________________________

Do not turn the page until you are told to begin. You will be given 100+ minutes to complete this exam. Show all your work on the exam itself; no credit will be given for anything written on other paper.

You may use a calculator.

Do not write in the following table; it will be used for grading.

Problem 1	___ / 10
Problem 2	___ / 15
Problem 3	___ / 30
Problem 4	___ / 30
Problem 5	___ / 15
Total	___ / 100

BASIC EQUATIONS OF ELECTRODYNAMICS

Maxwell's Equations

In General:

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{e_0} r$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{B} = m_0\vec{J} + m_0 e_0\frac{\partial\vec{E}}{\partial t}$$

In Matter:

$$\vec{\nabla}\cdot\vec{D} = r_f$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{H} = \vec{J}_f + \frac{\partial\vec{D}}{\partial t}$$

Auxiliary Fields

Definitions:

$$\vec{D} = e_0\vec{E} + \vec{R}$$

$$\vec{H} = \frac{1}{m_0}\vec{B} - \vec{M}$$

Linear Media:

$$\vec{R} = e_0 x_e\vec{E}, \quad \vec{D} = e\vec{E}$$

$$\vec{M} = x_m\vec{H}, \quad \vec{H} = \frac{1}{m}\vec{B}$$

Potentials

$$\vec{E} = -\vec{\nabla}V - \frac{\partial\vec{A}}{\partial t}, \quad \vec{B} = \vec{\nabla}\times\vec{A}$$

Lorentz Force Law

$$\vec{F} = q\left(\vec{E} + \vec{n}\ ^{\prime}\ \vec{B}\right)$$

Energy

Energy: $U = \frac{1}{2}\int\left(e_0 E^2 + \frac{1}{m_0}B^2\right)dt$

FUNDAMENTAL CONSTANTS

$\epsilon_0 = 8.85 \times 10^{-12}\, C^2/Nm^2$
(permittivity of free space)

$e = 1.60 \times 10^{-19}\, C$
(charge of the electron)

$\mu_0 = 4\pi \times 10^{-7}\, \mathrm{N/A^2}$
(permeability of free space)

$m = 9.11 \times 10^{-31}\, kg$
(mass of the electron)

SPHERICAL AND CYLINDRICAL COORDINATES

Spherical

$$x = r\sin\theta\cos\phi \qquad \hat{x} = \sin\theta\cos\phi\,\hat{r} + \cos\theta\cos\phi\,\hat{\theta} - \sin\phi\,\hat{\phi}$$
$$y = r\sin\theta\sin\phi \qquad \hat{y} = \sin\theta\sin\phi\,\hat{r} + \cos\theta\sin\phi\,\hat{\theta} + \cos\phi\,\hat{\phi}$$
$$z = r\cos\theta \qquad \hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$$

Cylindrical

$$x = s\cos\phi \qquad \hat{x} = \cos\phi\,\hat{s} - \sin\phi\,\hat{\phi}$$
$$y = s\sin\phi \qquad \hat{y} = \sin\phi\,\hat{s} + \cos\phi\,\hat{\phi}$$
$$z = z \qquad \hat{z} = \hat{z}$$

FUNDAMENTAL THEOREMS

Gradient Theorem: $\int_a^b (\nabla f)\cdot d\vec{l} = f(\vec{b}) - f(\vec{a})$

Divergence Theorem: $\int (\nabla \cdot \vec{A})\, d\tau = \oint \vec{A}\cdot d\vec{a}$

Curl Theorem: $\int (\vec{\nabla} \times \vec{A})\cdot d\vec{a} = \oint \vec{A}\cdot d\vec{l}$

VECTOR DERIVATIVES

Cartesian. $d\vec{l} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z};\ dt = dx\,dy\,dz$

Gradient: $\vec{\nabla} t = \frac{\partial t}{\partial x}\hat{x} + \frac{\partial t}{\partial y}\hat{y} + \frac{\partial t}{\partial z}\hat{z}$

Divergence: $\vec{\nabla} \cdot v = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$

Curl: $\vec{\nabla} \times v = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{x} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{y} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{z}$

Laplacian: $\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$

Spherical. $d\vec{l} = dr\,\hat{r} + r\,dq\,\hat{q} + r\sin q\,df\,\hat{j};\ \ dt = r^2 \sin q\,dr\,dq\,df$

Gradient: $\vec{\nabla} t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial q}\hat{q} + \frac{1}{r\sin q}\frac{\partial t}{\partial f}\hat{f}$

Divergence: $\vec{\nabla} \cdot v = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\right) + \frac{1}{r\sin q}\frac{\partial}{\partial q}\left(\sin q\, v_q\right) + \frac{1}{r\sin q}\frac{\partial v_f}{\partial f}$

Curl: $\vec{\nabla} \times v = \frac{1}{r\sin q}\left[\frac{\partial}{\partial q}\left(\sin q\, v_j\right) - \frac{\partial v_q}{\partial j}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin q}\frac{\partial v_r}{\partial j} - \frac{\partial}{\partial r}\left(r v_j\right)\right]\hat{q} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r v_q\right) - \frac{\partial v_r}{\partial q}\right]\hat{j}$

Laplacian: $\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin q}\frac{\partial}{\partial q}\left(\sin q\frac{\partial t}{\partial q}\right) + \frac{1}{r^2\sin^2 q}\frac{\partial^2 t}{\partial j^2}$

Cylindrical. $d\vec{l} = ds\,\hat{s} + s\,df\,\hat{f} + dz\,\hat{z};\ dt = s\,ds\,df\,dz$

Gradient: $\vec{\nabla} t = (\partial t/\partial s)\,\hat{s} + (1/s)(\partial t/\partial j)\,\hat{j} + (\partial t/\partial z)\,\hat{z}$

Divergence: $\vec{\nabla} \cdot v = \frac{1}{s}\frac{\partial}{\partial s}\left(s v_s\right) + \frac{1}{s}\frac{\partial v_f}{\partial f} + \frac{\partial v_z}{\partial z}$

Curl: $\vec{\nabla} \times v = \left[\frac{1}{s}\frac{\partial v_z}{\partial f} - \frac{\partial v_f}{\partial z}\right]\hat{s} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{f} + \frac{1}{s}\left[\frac{\partial}{\partial s}\left(s\, v_f\right) - \frac{\partial v_s}{\partial f}\right]\hat{z}$

Laplacian: $\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$

VECTOR IDENTITIES

Triple Products

(1) $\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$

(2) $\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$

Product Rules

(3) $\vec{\nabla}(fg) = f(\vec{\nabla}g) + g(\vec{\nabla}f)$

(4) $\vec{\nabla}(\vec{A}\cdot\vec{B}) = \vec{A}\times(\vec{\nabla}\times\vec{B}) + \vec{B}\times(\vec{\nabla}\times\vec{A}) + (\vec{A}\cdot\vec{\nabla})\vec{B} + (\vec{B}\cdot\vec{\nabla})\vec{A}$

(5) $\vec{\nabla}\cdot(f\vec{A}) = f(\vec{\nabla}\cdot\vec{A}) + \vec{A}\cdot(\vec{\nabla}f)$

(6) $\vec{\nabla}\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{A}) - \vec{A}\cdot(\vec{\nabla}\times\vec{B})$

(7) $\vec{\nabla}\times(f\vec{A}) = f(\vec{\nabla}\times\vec{A}) - \vec{A}\times(\vec{\nabla}f)$

(8) $\vec{\nabla}\times(\vec{A}\times\vec{B}) = (\vec{B}\cdot\vec{\nabla})\vec{A} - (\vec{A}\cdot\vec{\nabla})\vec{B} + \vec{A}(\vec{\nabla}\cdot\vec{B}) - \vec{B}(\vec{\nabla}\cdot\vec{A})$

Second Derivatives

(9) $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0$

(10) $\vec{\nabla}\times(\vec{\nabla}f) = 0$

(11) $\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A}$

$$\int \frac{x}{(a^2+x^2)^{\frac{3}{2}}}dx = \frac{-1}{\sqrt{a^2+x^2}}$$

$$\int \frac{x}{\sqrt{a^2+x^2}}dx = \sqrt{a^2+x^2}$$

$$\int \frac{\sin\vartheta\, d\vartheta}{\sqrt{a-\cos\vartheta}} = ln(a-\cos\vartheta)$$

$$\int \frac{x}{a^2+x^2}dx = \frac{1}{2}ln(a^2+x^2)$$

$$\int \frac{1}{\sqrt{a^2+x^2}}dx = \frac{1}{a}atan\left(\frac{x}{a}\right)$$

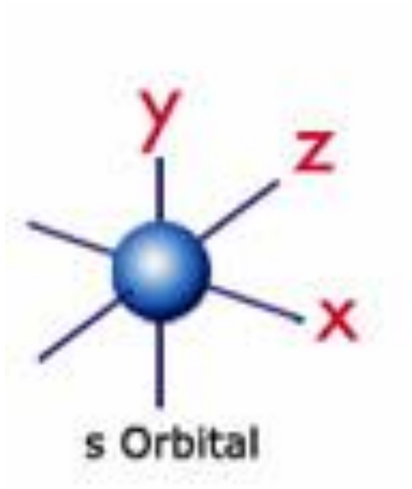
$$\int \frac{1}{(a^2+x^2)^{\frac{3}{2}}}dx = \frac{x}{a^2\sqrt{a^2+x^2}}$$

Problem 1: Fundamental Concept [10 pts] Why can I write a scalar potential for the electric field which is a vector?

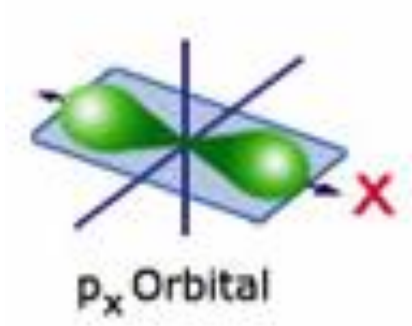
Problem 2: Multiple choice on an Upper level Exam?!? What? [15 pts] As you have learned (will learn) electron orbitals are important. The electron hangs around the nucleus in a cloud. We can find the charge density of the electron in a given state by using the following relationship:

$$\rho(r,\vartheta,\varphi) = -e\,\psi^{*}(r,\vartheta,\varphi)\psi\,(r,\vartheta,\varphi)$$

CIRCLE which of the following techniques CAN be directly used to find the electric field from the following charge distributions (there can be more than one method). Put a BOX around the method that is the BEST METHOD (only pick one).

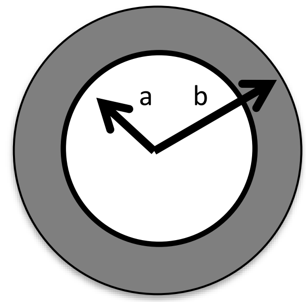


1. Discrete Coulomb's Law
2. Continuous Coulomb's Law
3. Gauss's Law
4. Method of images
5. Laplace Equation
6. Multiple expansion



1. Discrete Coulomb's Law
2. Continuous Coulomb's Law
3. Gauss's Law
4. Method of images
5. Laplace Equation
6. Multiple expansion

Problem 3: You'd think the bad guys would have learn Death Stars Don't Work [30 pts]: While constructing the Death Star, there is a large, solid but hollow conducting shell. It has inner radius a, and outer radius b. (The inside is completely empty of matter.) Cosmic rays have charged the shell to a total negative charge, –q. No other charges are around; we're in equilibrium.



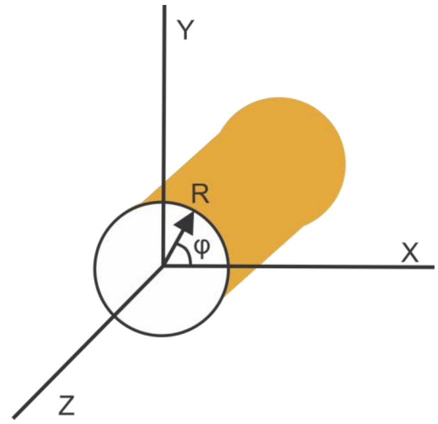
(a) [6 pts] Write out the charge distribution anywhere it's not zero.
(b) [8 pts] Find and sketch the radial component of the E-field as a function of radius. Make sure to annotate the graph completely.
(c) [8 pts] Find and sketch the voltage as a function of radius (assuming $(\infty) \to 0$). Make sure to annotate the graph completely.
(d) [8 pts] How much electrostatic energy is built up in the system?

Problem 3: Con't

Problem 3: Con't

Problem 3: Con’t

Problem 4: Only half! [30 pts]: The potential at a surface of an infinite cylinder of radius R is given by:



$$V(R,\varphi) = \begin{cases} V & 0 \le \varphi < \pi \\ 0 & \pi \le \varphi < 2\pi \end{cases}$$

(where V is a constant). Find the potential inside and outside of the cylinder. There is no need to solve the differential equation, the solution is below:

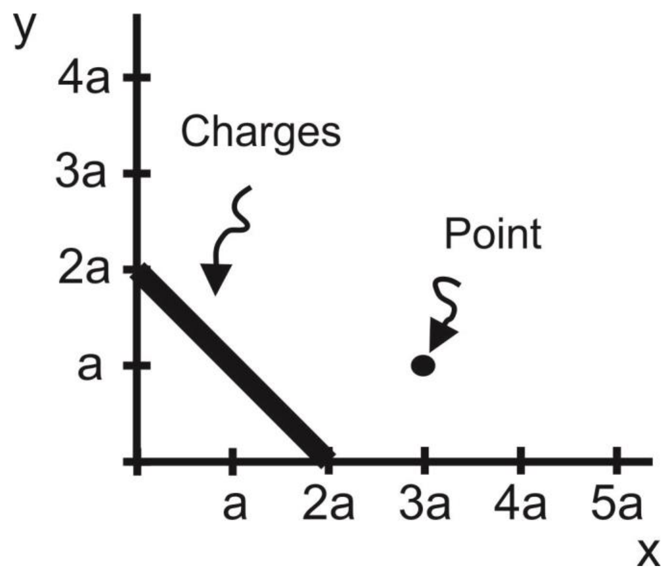
$$V(s,\varphi) = b_0 \ln s + \sum_{n=-\infty\ (n\neq 0)}^{\infty} s^n (A_n \sin n\varphi + B_n \cos n\varphi)$$

Problem 4: Con’t

Problem 4: Con't

Problem 4: Con’t

Problem 5: Hey I've done this before … [15 pts]: You have a short (finite!) uniform line charge λ as shown below. Calculate the electric field at point P.



Problem 5: Con’t