

Physics 380-001 Homework

1. **Hydrogen (2 pts):** Do problem 4.2 in textbook

2. **Energy (2 pts):** Do problem 4.7 in textbook

3. **Now I'm stuck in a pipe (6 pts):** A sphere of radius R carries a polarization of

$$\vec{P} = kr^2\hat{z}$$

- What are the units of k ?
- Calculate the bound charges σ_b and ρ_b
- In your own words describe how you'd find the field everywhere. If you want to actually do it, go for it!

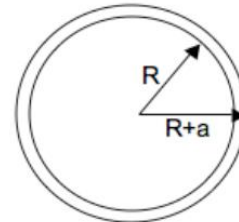
4. **Strange pipe (2 pts):** Do problem 4.13 in textbook

5. **Gauss's Law Again (2 pts):** Do problem 4.15 in textbook

6. **Linear Yo! (2 pts):** Do problem 4.18 in textbook

7. **Ughness! (2 pts):** Do problem 4.24 in textbook

8.) The axon (8 pts). Each neuron has an axon, which extends from the cell body to the synaptic terminus where it makes contact with another neuron or with some other kind of cell (e.g. a muscle cell). The longest axons in your body stretch roughly 1 m from the base of your spine to your toes.



- We can treat an axon as a long cylinder with inner radius R and outer radius $R + a$, as shown in cross-section in the figure. The inner layer carries a charge per unit length of $-\lambda$, and the outer layer $+\lambda$. Between these charged layers of ions, there is a lipid layer with dielectric constant κ . Use Gauss's Law to calculate the radial electric field E_r as a function of radius r between everywhere (including in-between the layers in the axon). Express your answer in terms of r , R , λ , a , ϵ , and fundamental constants.
- Using the expression for E_r you found in part (a) above, find an expression for the potential everywhere.
- Now consider an axon 1 m long with an inner radius of $1.5 \mu\text{m}$. If this axon has a typical cell membrane with a thickness of 6 nm and a dielectric constant of $9 (\epsilon/\epsilon_0)$, find the capacitance of this axon.
- Most neurons have more than just a simple cell membrane—they are also surrounded by a thick sheath of myelin. If the same neuron described in part (c) above adds a myelin sheath with a thickness of $2 \mu\text{m}$, find the capacitance of the myelinated neuron. (The dielectric constant of

myelin is also approximately 9.) Does the myelin sheath cause the capacitance to increase or decrease?

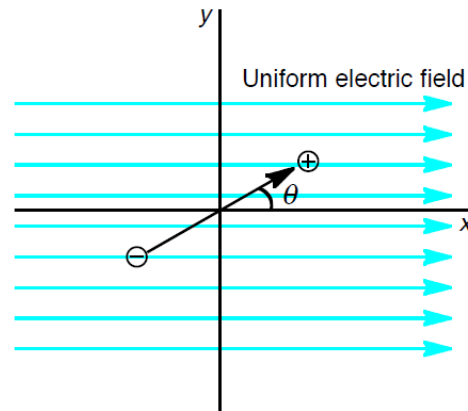
9.) Capacitors Again Again (4 pts): Please do Problem 4.20 in Text Book

10.) Dipoles (8 pts):

- Please do problem 4.5 in Text Book
- Please do problem 4.29 in Text Book

11.) Dipoles for kindergarteners (8 pts):

Consider a dipole with charges $\pm q$ separated by a distance d . The center of the dipole is located at the origin, and there is a uniform electric field E that points in the positive x -direction, as shown in the figure.



- Using the definition of dipole moment p that we discussed in class, show that the magnitude of the torque on this dipole is given by $E p \sin \theta$, where θ is the angle between the dipole moment vector \vec{p} (shown on the figure) and the electric field vector. (Be careful about pesky factors of 2!)
- Find an expression for $V(x)$, the potential as a function of x , from the uniform electric field (ignoring the dipole). Take the potential $V = 0$ where $x = 0$.
- Show that the potential energy of the dipole, as a function of θ , is given by $U = -\vec{p} \cdot \vec{E}$, if you take the potential energy equal to zero when $\theta = 90^\circ$.
- Sketch the configurations where the potential energy is at a maximum, and where it is at a minimum. What is the torque on the dipole in each configuration? If the torque is zero for a particular configuration, is that a point of stable or unstable equilibrium? Show a configuration for which the magnitude of the torque is at a maximum.