Problem 1: A sphere of radius R carries a polarization $\vec{P} = k\vec{r}$, where k is a constant.

a. (Concept) Calculate the bound charges σ_b and ρ_b.
b. Find the field inside and outside of the sphere.

Remember so definitions:

Bound charge density:

$$\rho_b = -\vec{\nabla} \cdot \vec{P}$$

Where ρ_b is the bound charge density and $\vec{P}$ is the polarization (dipoles per unit volume)

Surface Bound charge density:

$$\sigma_b = \vec{P} \cdot \hat{n}$$

Where σ_b is the surface bound charge density and $\hat{n}$ is the normal unit vector (ie, perpendicular to the surface)

So if $\vec{P} = k\vec{r}$:

Which coordinate system should you use to find ρ_b and σ_b?

Sketch what this looks like (in terms of dipoles):

Find ρ_b and σ_b

Is there any symmetry in the problem? Identify it if there is?

Will this symmetry allow you to use Gauss's law?

How many regions are there?

Solve?

Problem 2: Find the electric field produced by a uniformly polarized sphere of radius R.

So this problem is tricky:

$$\vec{P} = P_0\hat{k}$$

Sketch this?

Find ρ_b and σ_b. Don't be afraid to be creative in thinking about the coordinate system choice:

Is there any symmetry in the problem? Identify it if there is?

So:

$$\sigma_b = P_0 \cos\vartheta \qquad\qquad and \qquad\qquad \rho_b = 0$$

So you know that Laplace's equation holds:

$$\nabla^2 V = -\frac{\rho}{\epsilon_0} = -\frac{1}{\epsilon_0}(\rho_f + \rho_b) \to 0$$

And you can write the surface charge density as:

$$\sigma = \sigma_f + \sigma_b = 0 + P_0 \cos\vartheta = P_0 P_1(\cos\vartheta)$$

Use context clues to identify what the differences between P_0 and $P_1(\cos\vartheta)$ are:

What is the general solution for Laplace's equation in spherical coordinates?

Apply boundary conditions at $r \to 0$ and $r \to \infty$. How does that change the potential equation in the different regions?

Now identify what the boundary conditions are at the surface (i.e., $r \rightarrow R$). Hint, what does the surface charge density say about potential at the boundary?

As you expand your Legendre polynomials, you remember that the Legendre series is orthogonal and complete and the equation must hold for all values of ϑ. It seems like there is a 'Trick' or two you can use. What is the game plan?

Solve: