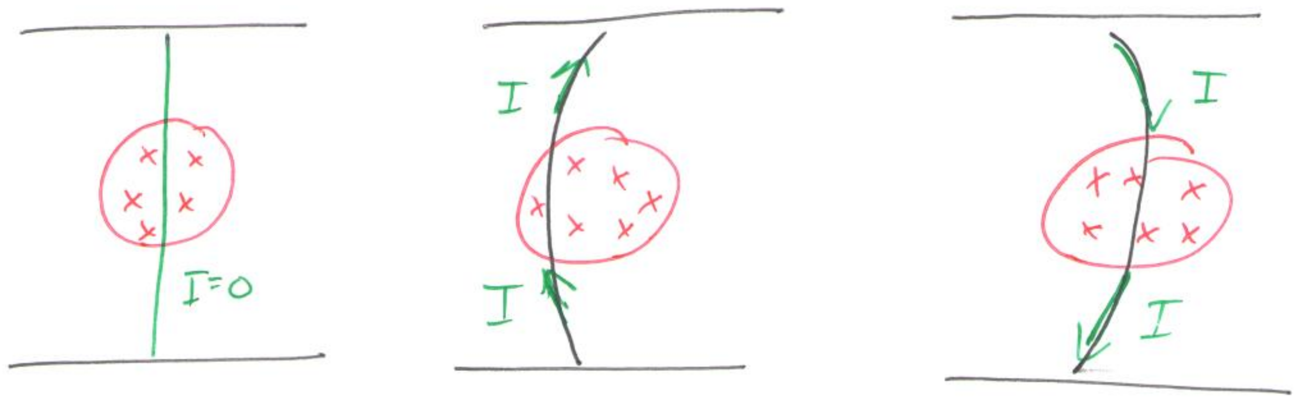
Problem 1



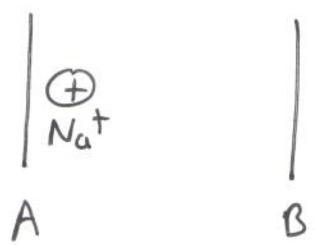
Problem 6

Part a)

$\Delta V = 1\,kV = 1.6 \times 10^{-16}\,J$

$Na \Rightarrow m = 23 \times 1.67 \times 10^{-27}\,kg$

$= 3.8 \times 10^{-26}\,kg$



Energy must be conserved

$$E_0 = E_f = \underbrace{\tfrac{1}{2} m v^2}_{\text{Final}} + 0 \leftarrow \text{Potential}$$

initialy the ion is at rest $\quad E_0 = \cancel{\tfrac{1}{2} m v_0^2} + P.E. = \underline{\underline{qV}} \quad q \to e^+$

$$\boxed{\text{So A must be } qV \text{ whereas B is } 0V}$$

$$E_0 = qV = \tfrac{1}{2} m v^2 \Rightarrow m v^2 = 2qV \qquad \boxed{v = 92\ \tfrac{km}{s}}$$

Part b

circular motion: The constraint is $a_c = v^2/r$

$$F = m\frac{v^2}{r} = ma$$

$F \Rightarrow r = 10\,cm = 0.1\,m \Rightarrow$

$$F = \frac{2qV}{r} = \frac{2 \times e \times 1\,kV}{0.1\,m}$$

$$\boxed{F = 3.2 \times 10^{-15}\,N}$$

Part c)

$$\vec{F} = q\,\vec{v} \times \vec{B}$$

Points down "Thumb down"

Points to the right on the page. "Index"

middle finger points out of page!

$$F = qvB$$

$$\frac{2qV}{r} = qvB$$

$$r = \frac{2V}{vB} \Rightarrow \boxed{B = \frac{2V}{vr} = 0.22\,T}$$

Part d)

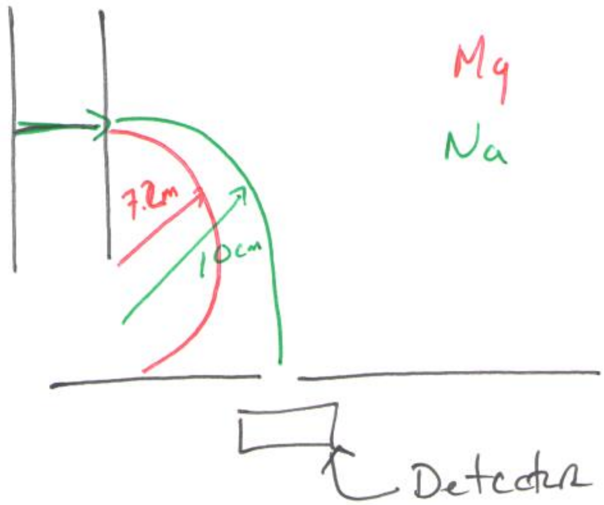
Magnesium $q = 2e$ $M = 24 \times 1.67 \times 10^{-27}\,kg = 4.0 \times 10^{-26}\,kg$

$$\frac{v_{Na}}{v_{Mg}} = \sqrt{\frac{q_{Na}/M_{Na}}{q_{Mg}/M_{Mg}}} = \sqrt{\frac{q_{Na}}{q_{Mg}}\frac{M_{Mg}}{M_{Na}}} = \sqrt{\frac{1}{2} \cdot \frac{24}{23}} = 0.72$$

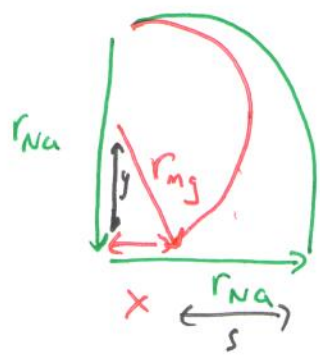
$$v_{Mg} = 128\ km/s$$

$$r = \frac{2V}{vB} \Rightarrow \frac{r_{Mg}}{r_{Na}} = \frac{v_{Na}}{v_{Mg}} = \frac{92}{128} = 0.72$$

$$\boxed{r_{Mg} = 7.2\ cm}$$



We are going to have to move the detector to the left



$$y^2 + x^2 = r_{mg}^2$$

$$(r_{Na} - r_{mg})^2 + x^2 = r_{mg}^2$$

$$x = \sqrt{r_{mg}^2 - (r_{Na} - r_{mg})^2}$$

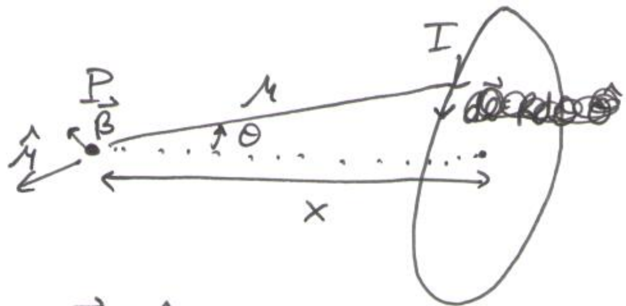
$$S = r_{Na} - x$$

$$S = r_{Na} - \sqrt{r_{mg}^2 - (r_{Na} - r_{mg})^2}$$

$\boxed{S = 3.1 \text{ cm to the left.}}$

Problem 3

Part a.)



$$\vec{B} = \frac{\mu_0}{4\pi} I \int \frac{\vec{dl} \times \hat{r}}{r^2}$$

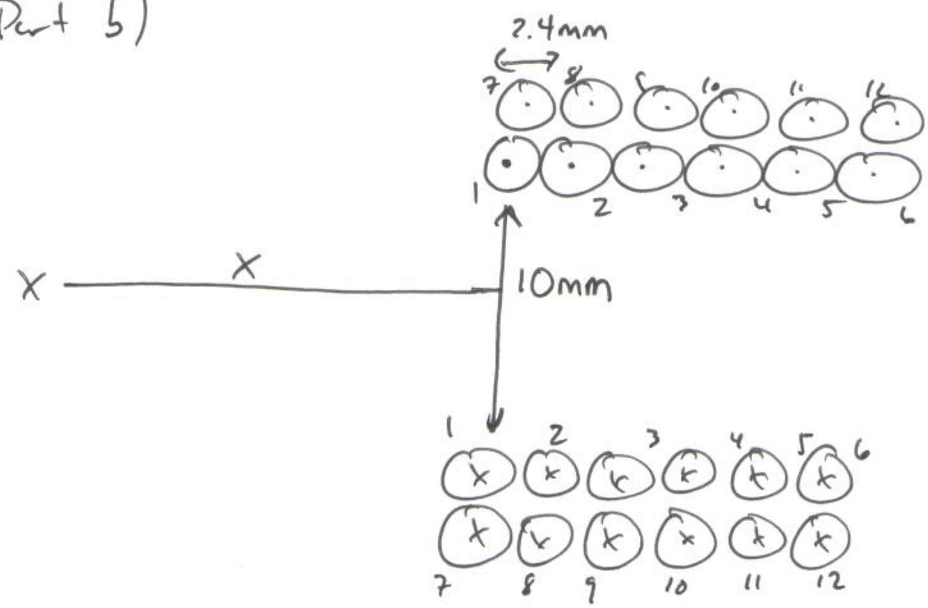
As I add up around the loop only the $\hat{z}$-direction survives

$$\vec{dl} \times \hat{r} = dl \cos\theta = R\,d\phi \cos\theta$$

$$\oint \vec{dl} \times \hat{r} = 2\pi R \cos\theta$$

$$\vec{B} = \frac{\mu_0}{4\pi} I \frac{2\pi R \cos\theta}{(R^2 + x^2)} \hat{z} = \frac{\mu_0 I R}{2} \frac{\cos\theta}{(R^2+x^2)} \hat{z} = \frac{\mu_0 I R^2}{2} (R^2+x^2)^{-3/2} \hat{z}$$

Part b)



The first coil (1) has an

$x_1 = x$

$R_1 = 5\text{mm} + \frac{w}{2}$

$R_1 = 5\text{mm} + 1.2\text{mm}$

$R_1 = 6.2\text{mm}$

coil 2 : x has increased by one width (of the wire)

$$X_2 = X + w = X + 2.4\,mm$$

$$R_2 = 6.2\,mm = R_1$$ ⇐ hasn't changed

Coil 3:

$$x_3 = X_2 + w = X + 2.4 + 2.4 = x + 4.8\,mm$$

$$R_3 = R_2 = R_1$$

⋮

Coil 7: $X_7 = X_1 = X$ ⇐ Start over

$$R_7 = R_1 + w = 6.2\,mm + 2.4\,mm = 8.6\,mm$$

⋮

SEE The Calculation on the next Page

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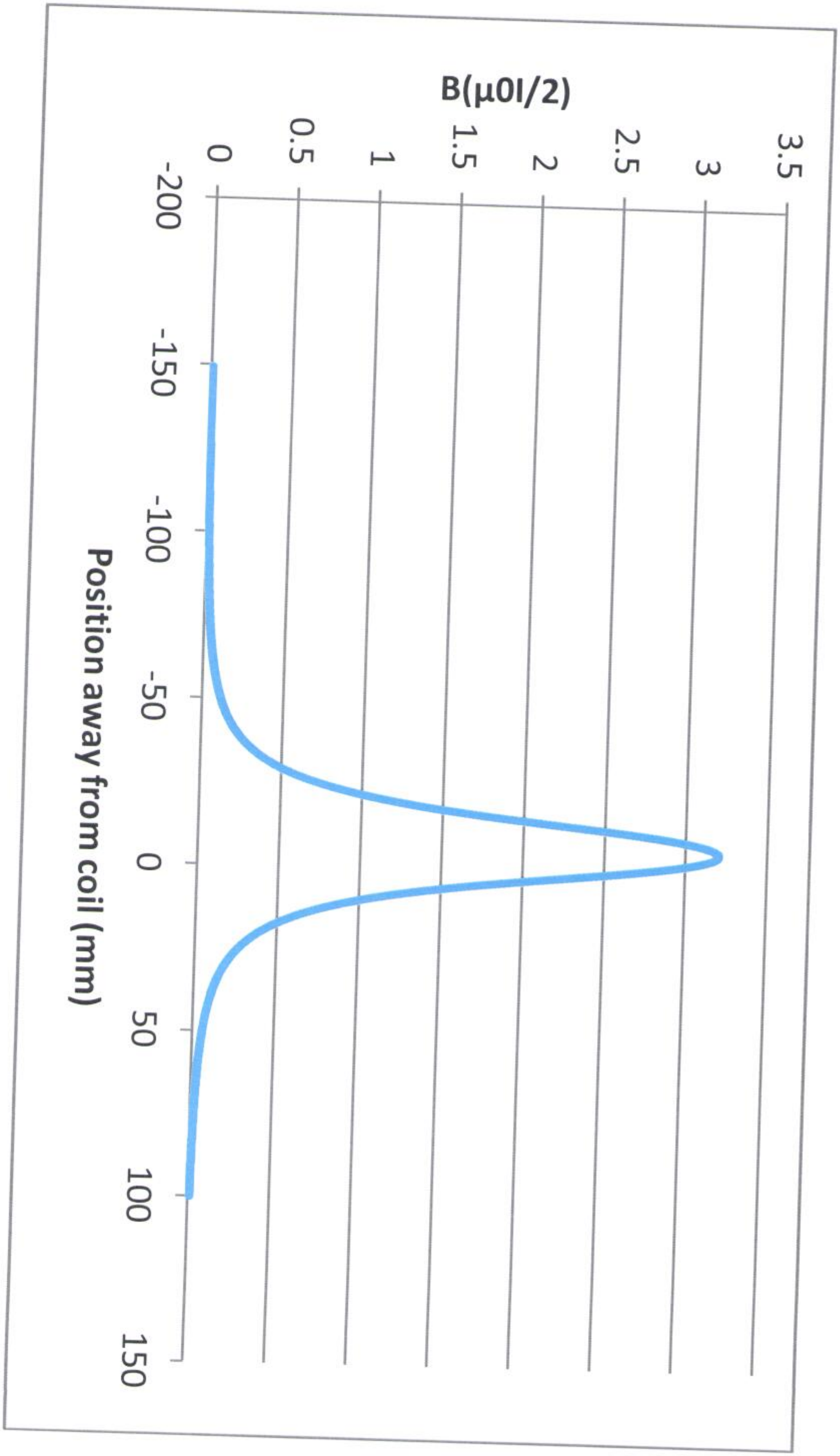
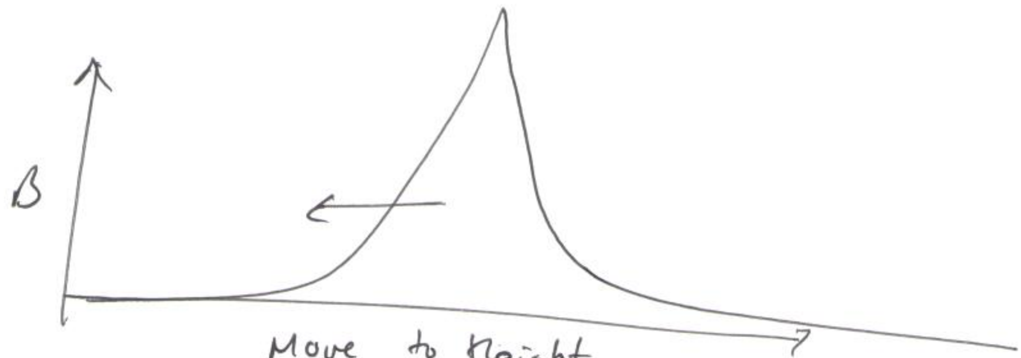
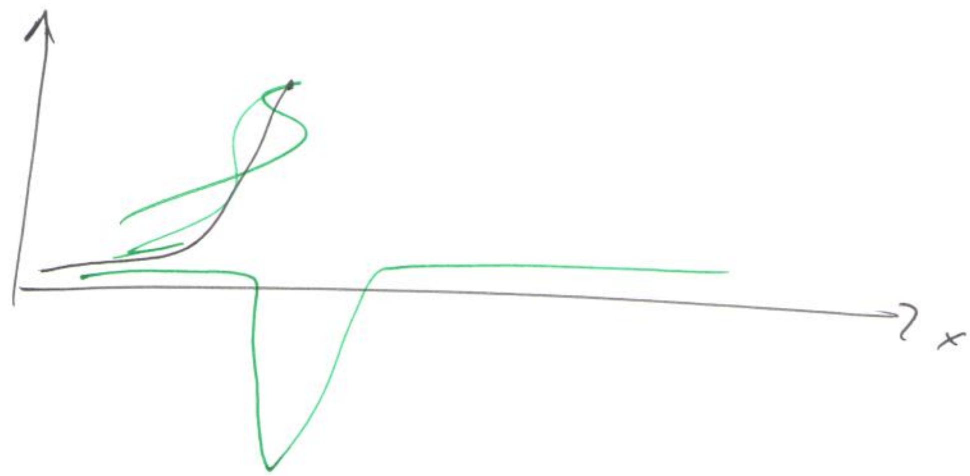


B(µ0I/2)
3.5
3
2.5
2
1.5
1
0.5
0
-200
-150
-100
-50
0
50
100
150
Position away from coil (mm)



Move to the right
by 27 mm & multiply by −1



Then add them & square

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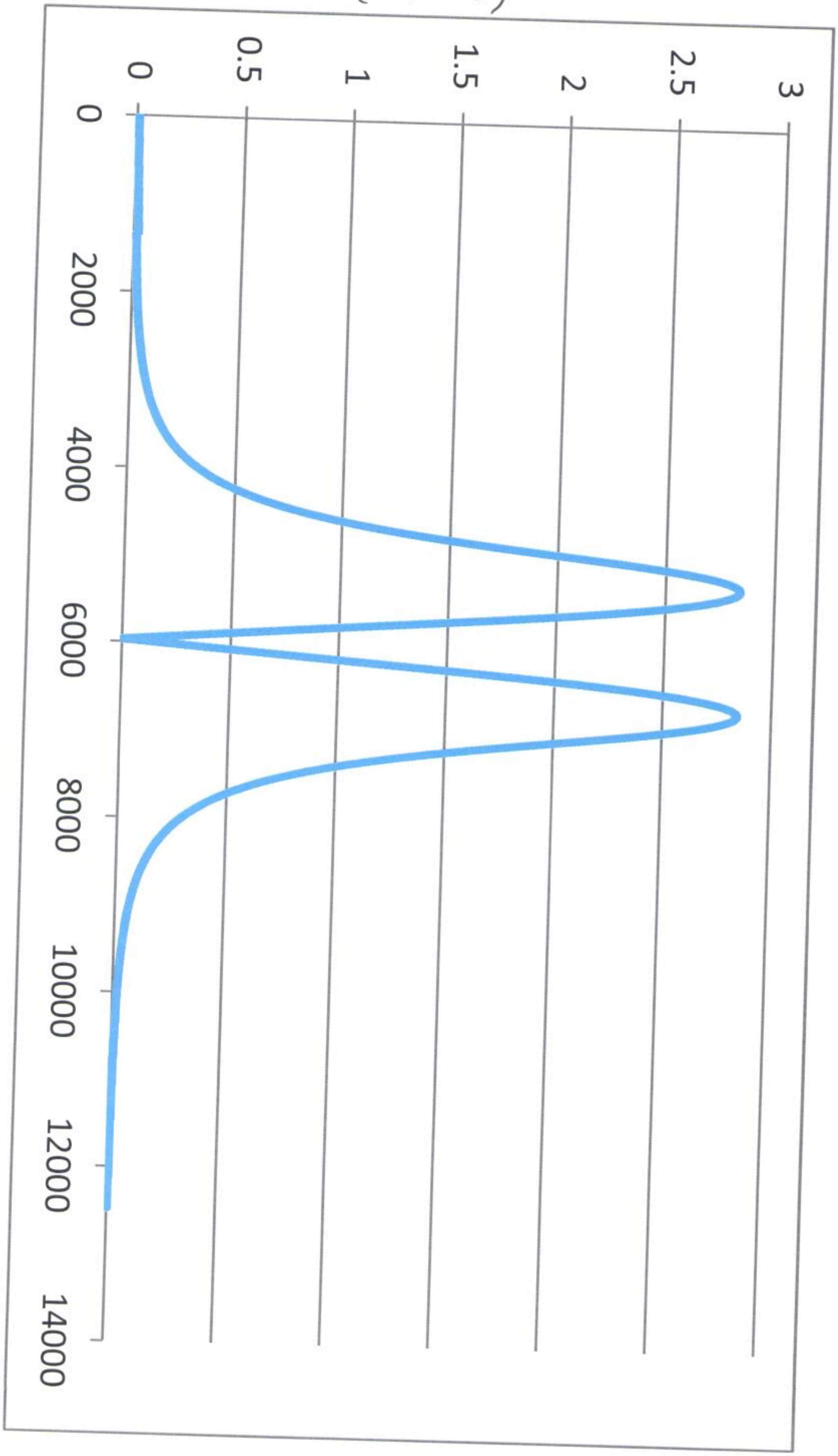
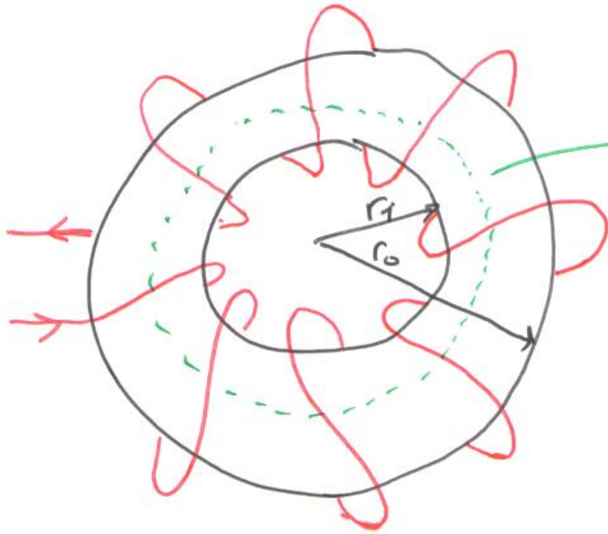
Abs $(B_1 - B_2)$



$P_{1,s}$

Problem 4



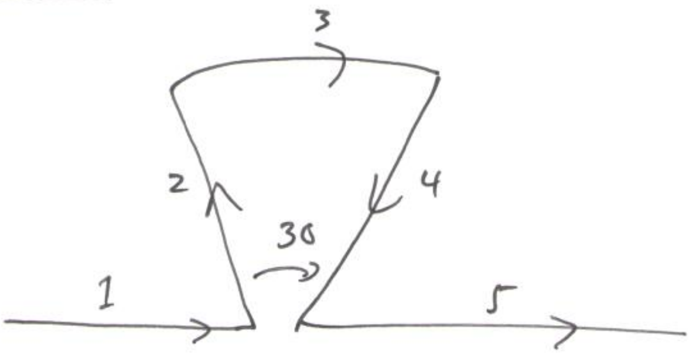
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

$$B \cdot 2\pi r = \mu_0 N I$$

# of coils

$$B = \frac{\mu_0 N I}{2\pi r} \text{ from } r_i \text{ to } r_o$$

Problem 5



$$\vec{B} = \int \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\left[\int_1 \frac{d\vec{l}\times\hat{\mathscr{r}}}{\mathscr{r}^2} + \int_2 \frac{d\vec{l}\times\hat{\mathscr{r}}}{\mathscr{r}^2} + \int_3 \frac{d\vec{l}\times\hat{\mathscr{r}}}{\mathscr{r}^2} + \int_4 \frac{d\vec{l}\times\hat{\mathscr{r}}}{\mathscr{r}^2} + \int_5 \frac{d\vec{l}\times\hat{\mathscr{r}}}{\mathscr{r}^2}\right]$$

1: $\hat{\mathscr{r}} = \hat{\imath}$, $dl = dx\,\hat{\imath}$, $\hat{\imath}\times\hat{\imath} = 0$

2: $\hat{\mathscr{r}} = \hat{r}$, $d\vec{l} = dr\,\hat{r}$, $\hat{r}\times\hat{r} = 0$

3: $\mathscr{r}^2 = r^2$, $\hat{\mathscr{r}} = \hat{r}$, $d\vec{l} = r\,d\theta\,\hat{\theta}$

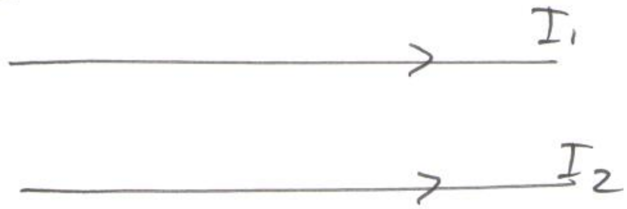
4: $\hat{\mathscr{r}} = -\hat{r}$, $d\vec{l} = dr\,\hat{r}$, $\hat{r}\times\hat{r} = 0$

5: $\hat{\mathscr{r}} = -\hat{r}$, $d\vec{l} = dr\,\hat{r}$, $\hat{r}\times\hat{r} = 0$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^{30^\circ} \frac{r\,d\theta}{r^2} (\hat{\theta} \times \hat{r})$$

$$\vec{B} = \frac{\mu_0 I}{4\pi r} \underbrace{(-\hat{k})}_{\text{Into the page}} \int_0^{\pi/6} d\theta = \frac{\mu_0 I}{24R} (-\hat{k})$$

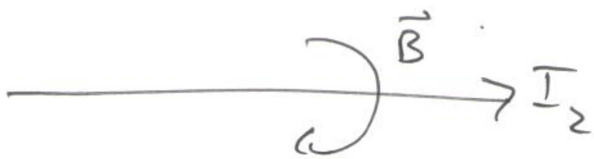
Problem 6



$$\vec{F} = I \int d\vec{s} \times \vec{B} = I \int d\vec{\ell} \times \vec{B}$$

$$\vec{F}_{12} = I_1 \int d\vec{\ell} \times \vec{B}_2$$

USE Ampere's Law to Find B_2



$$\int \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$$

$$2\pi r B = \mu_0 I_2$$

$$B_2 = \frac{\mu_0 I_2}{2\pi r}$$

$$\vec{B}_2 = \frac{\mu_0 I_2}{2\pi r} \hat{\theta}$$

The magnetic Field at wire 1 due to wire 2 is just pointing in the $\hat{k}$ direction assuming $d\vec{l}$ points in $\hat{i}$

$$\vec{F} = I_1 \int d\vec{l} \times \vec{B_2}$$

$$\vec{F} = I_1 \int dx\, \hat{i} \times \frac{\mu_0 I_2}{2\pi r} \hat{k}$$

$$\vec{F} = \frac{\mu_0 I_1 I_2}{2\pi r} \int dx\, (\hat{i} \times \hat{k})$$

$$\vec{F} = -\frac{\mu_0 I_1 I_2}{2\pi r} l\, \hat{j}$$

Problem 7

$$\vec{A} = -\frac{1}{2}(\vec{r} \times \vec{B})$$

$$\vec{\nabla} \cdot \vec{A} = \vec{\nabla} \cdot \left[-\frac{1}{2} \vec{r} \times \vec{B}\right] = -\frac{1}{2} \vec{\nabla} \cdot \epsilon_{ijk}\, \hat{i}\, r_j B_k$$

$$= -\frac{1}{2} \epsilon_{ijk}\, \partial_i r_j B_k$$

Product rule

$$= -\frac{1}{2} \epsilon_{ijk} \left[\left(\frac{\partial}{\partial x_i} x_j\right) B_k + x_j \frac{\partial}{\partial x_i} B_k\right]$$

$\frac{\partial x_j}{\partial x_i} = \delta_{ij}$

B is a constant

0

$$= -\frac{1}{2} \epsilon_{ijk}\, \delta_{ij} B_k$$

$$= -\frac{1}{2} \epsilon_{iik}\, B_k = 0$$

0

✓

$\vec{\nabla} \times \vec{A} = \vec{B}$

$$\vec{B} = \vec{\nabla} \times \left(-\frac{1}{2}\, \vec{r} \times \vec{B} \right)$$

$$\vec{B} = \vec{\nabla} \times -\frac{1}{2} \epsilon_{ijk}\, \hat{i}\, r_j B_k$$

$$\vec{B} = \epsilon_{mni}\, \partial_n\, \epsilon_{ijk}\, r_j B_k\, \hat{m} \left(-\tfrac{1}{2}\right)$$

$$\vec{B} = \epsilon_{mni} \epsilon_{ijk}\, \underbrace{\partial_n r_j}\, B_k\, \hat{m} \left(-\tfrac{1}{2}\right)$$

$$\vec{B} = \epsilon_{mni} \epsilon_{ijk}\, \delta_{nj}\, B_k\, \hat{m} \left(-\tfrac{1}{2}\right)$$

$$\vec{B} = \epsilon_{ijk} \epsilon_{inm} \left(+\tfrac{1}{2}\right) \delta_{nj}\, B_k\, \hat{m}\, \cancel{\left(-\tfrac{1}{2}\right)}$$

$$\vec{B} = (\delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km}) \left(+\tfrac{1}{2}\, \delta_{nj}\, B_k\, \hat{m}\right)$$

$$= (3\delta_{jm}\delta_{kj} - \delta_{jj}\delta_{km}) \left(+\tfrac{1}{2}\, B_k\, \hat{m}\right)$$

$$= [3\delta_{mk} - \delta_{km}] \left(+\tfrac{1}{2}\, B_k\, \hat{m}\right)$$

$$= 2\delta_{mk} \left(\tfrac{1}{2}\right) B_k\, \hat{m} = B_k\, \hat{k} = \vec{B}$$

✓

Problem 11

Problem 5.14 in text

Part a.)

← Current on the outside only

← inside Amperian loop

$$\int \vec{B}_{in} \cdot d\vec{l} = \mu_0 I_{enc} = 0$$

$$\boxed{B_{in} = 0}$$

← Amperian loop outside

$$\int \vec{B}_{out} \cdot d\vec{l} = \mu_0 I_{enc} = \mu_0 I$$

$$2\pi s B_0 = \mu_0 I$$

$$\boxed{B_0 = \mu_0 I / 2\pi s}$$

Part b.) $\vec{J} = \alpha s \hat{k}$

$$I = \int \vec{J} \cdot d\vec{a} = \int J_k \, da_k = \int J_k \, s \, ds \, d\varphi = \int_0^a \alpha s \cdot s \, ds \, d\varphi$$

$$I = 2\pi\alpha \int_0^a s^2 ds = \frac{2\pi\alpha}{3} a^3$$

$$\alpha = \frac{3I}{2\pi a^3} \Rightarrow \vec{J} = \frac{3Is}{2\pi a^3} \hat{k}$$

inside

$$\oint B \cdot dl = \mu_0 I_{enc}$$

$$2\pi s B = \mu_0 \int_0^s \vec{J} \cdot d\vec{a}$$

$$2\pi s B = \mu_0 \frac{3I}{2\pi a^3} \int_0^s s \cdot s \, ds \, d\phi$$

$$2\pi s B = \frac{\mu_0 I}{a^3} s^3 \Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi a^3} s^2}$$

outside

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi s}}$$