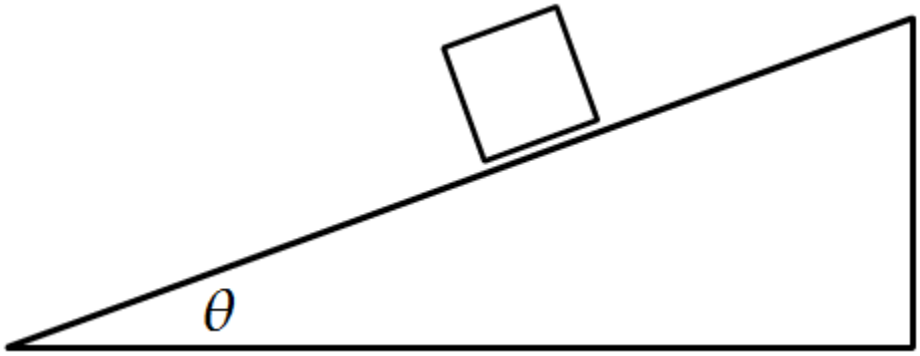
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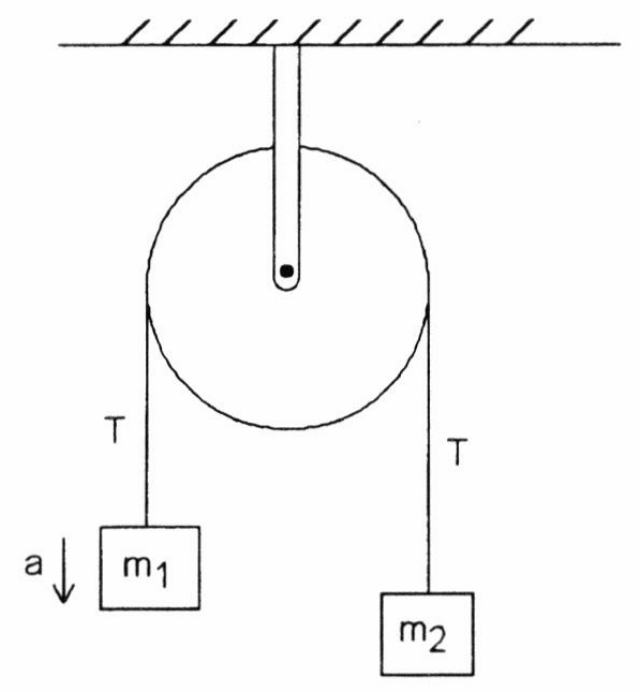
Homework #4: Calculus of Variations II

(1) Use the Lagrangian to find the equations of motion of a block sliding down an inclined plane.



(2) Consider a particle of mass m moving in a plane subject to an inverse square attractive force. Obtain the equations of motion using the Lagrangian.

(3) Atwood's machine: Consider an Atwood machine consisting of a single pulley of moment of inertia I about an axis through its center and perpendicular to its plane. The length of the inextensible connecting the two masses and going over the pulley is ℓ. Calculate the acceleration of the system. The string is assumed massless and inextensible, and the pulley is frictionless.



(4) An inclined plane of mass M is sliding on a smooth horizontal surface, while a particle of mass m is sliding on a smooth included surface. Find the equations of motion of the particle and the inclined plane.

(5) A. Write the Hamiltonian for a simple pendulum and derive its equation of motion.

B. Using Matlab Numerically solve for the trajectory of the pendulum. Discuss how good of an approximation $\sin\vartheta \sim \vartheta$ and when it starts to break down.

(6) An oscillating pendulum, consists of a mass m attached to a massless spring of spring constant k and of relaxed length ℓ_0. Find Lagrenge's equations of motion. Under what conditions will the system reduce to a simple pendulum and under what conditions will the system reduce to a linear harmonic oscillator.