Power Series Tutorial

You!

March 21, 2017

Consider the following differential equation (note α is a constant):

$$\dot{x} = \alpha x \tag{1}$$

How would you solve this equation?

What is the solution?

There are many different ways to solve differential equations, including direction integration, guessing, change of variables, Using the quadratic equation, Laplace Transforms, and my personal favorite Green's Functions. Here we are going to learn a new way to solve differential equations. We are doing do it with a series solution.

It starts with kind of a guess. We are gong to assume that the solution of the following equation is of the form:

$$x(t) = \sum_{n=0}^{\infty} a_n t^n \tag{2}$$

What is the time derivative of equation (2)? Make sure to consider how that might be different for different values of n.

Next plug equations (3) and (2) into equation (1). This vary complicated equation must be true for all values of t. That means that all the coefficients of powers of t must solve independent equations. Find how a_1 depends on a_0. How does a_2 depend of a_1? What is the general solution for a_{n+1} in terms of a_n?

Now write out the series solution for the first few terms. Discuss is this the same or different the result you found by solution equation (1) the first time?

Now solve the following equation two different ways:

$$y' = 2xy \tag{3}$$